Soit $n \in \mathbb{N}^*$, un entier naturel, alors :

$$n! = n \times (n-1) \times \ldots \times 3 \times 2 \times 1$$

La factorielle de n peut aussi s'exprimer comme :

$$n! = 2^{\left(\sum_{\alpha=0}^{\alpha=\lfloor \log_2 n \rfloor} \alpha\left(\left\lfloor \frac{n-2^{\alpha}}{2^{\alpha+1}} \right\rfloor + 1\right)\right)} \prod_{\alpha=0}^{\alpha=\lfloor \log_2 n \rfloor} \left(\prod_{k=\left\lfloor \frac{n-2^{\alpha+1}}{2^{\alpha+2}} \right\rfloor + 1}^{k=\left\lfloor \frac{n-2^{\alpha}}{2^{\alpha+1}} \right\rfloor} (2k+1) \right)^{\alpha+1}$$

$$n! = 2^{\left(\sum_{\alpha=0}^{\alpha=\lfloor \log_2 n \rfloor} \alpha\left(\left\lfloor \frac{n-2^{\alpha}}{2^{\alpha+1}} \right\rfloor + 1\right)\right)} 3^{\left(\sum_{\beta=0}^{\beta=\lfloor \log_3 n \rfloor} \beta\left(\left\lfloor \frac{n-3^{\beta}}{3^{\beta+1}} \right\rfloor + 1\right)\right)} \prod_{\alpha=0}^{\alpha=\lfloor \log_2 n \rfloor} \prod_{\beta=0}^{\beta=\lfloor \log_3 n \rfloor} \left(\left(\prod_{k=\left\lfloor \frac{n-2^{\alpha+1}3^{\beta+1}}{2^{\alpha+2}3^{\beta+2}} \right\rfloor + 1}^{k=\left\lfloor \frac{n-2^{\alpha}3^{\beta}}{2^{\alpha+1}3^{\beta+1}} \right\rfloor} (6k+1) \right) \left(\prod_{k=\left\lfloor \frac{n-2^{\alpha+1}3^{\beta+1}5}{2^{\alpha+2}3^{\beta+2}} \right\rfloor + 1}^{k=\left\lfloor \frac{n-2^{\alpha}3^{\beta}5}{2^{\alpha+1}3^{\beta+1}} \right\rfloor} (6k+5) \right) \right)^{\alpha+\beta+1}$$

Soit $\omega \in \mathbb{P}$, un nombre premier, a fortiori :

$$n! = \left(\prod_{\substack{p \in \mathbb{P} \\ p \leqslant \omega}} \left(p^{\left(\sum_{\alpha_p=0}^{\alpha_p=\lfloor \log_p n \rfloor} \alpha_p\left(\left\lfloor \frac{n-p^{\alpha_p}}{p^{\alpha_p+1}} \right\rfloor + 1\right)\right)} \right) \right) \left(\prod_{\alpha_2=0}^{\alpha_2=\lfloor \log_2 n \rfloor} \prod_{\alpha_3=0}^{\alpha_3=\lfloor \log_3 n \rfloor} \prod_{\alpha_5=0}^{\alpha_5=\lfloor \log_5 n \rfloor} \cdots \prod_{\alpha_\omega=0}^{\alpha_\omega=\lfloor \log_\omega n \rfloor} \left(\prod_{\left\{\substack{p \in \mathbb{P} \\ p > \omega \\ p < \prod_{\substack{q \in \mathbb{P} \\ q \leqslant \omega}} q}\right\} \cup \{1\}} \left(\prod_{k=\left\lfloor \frac{n-p \prod_{\substack{q \in \mathbb{P} \\ q \leqslant \omega}} q^{\alpha_q+1}}{\prod_{\substack{q \in \mathbb{P} \\ q \leqslant \omega}} q^{\alpha_q+2}} \right\rfloor + 1}^{k=\left\lfloor \frac{n-p \prod_{\substack{q \in \mathbb{P} \\ q \leqslant \omega}} q^{\alpha_q}}{\prod_{\substack{q \in \mathbb{P} \\ q \leqslant \omega}} q^{\alpha_q+1}} \right\rfloor} \left(\left(\prod_{\substack{q \in \mathbb{P} \\ q \leqslant \omega}} q \right) k + p \right) \right) \right)^{1+\sum_{\substack{q \in \mathbb{P} \\ q \leqslant \omega}} \alpha_q} \right)$$

Amusement de Kevin Trancho